

Color Superconducting Quark Matter in Neutron Stars

Henning Heiselberg

NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Color superconductivity in quark matter is studied for electrically charge neutral neutron star matter in β -equilibrium. Both bulk quark matter and mixed phases of quark and nuclear matter are treated. The electron chemical potential and strange quark mass affect the various quark chemical potentials and therefore also the color superconductivity due to dicolor pairing or color-flavor locking.

Strongly interacting matter is expected to undergo a transition to chirally restored matter of quarks and gluons at sufficiently high baryon or energy density. Such phase transitions are currently investigated in relativistic heavy-ion collisions and may exist in the interior of neutron stars. At low temperatures a condensate of quark Cooper pairs may appear characterized by a BCS gap Δ [1–5] usually referred to as color superconductivity (CSC). The appearance of a gap through color-flavor locking (CFL) requires the gap to exceed the difference between the quark Fermi momenta, which is not the case for sufficiently large strange quark masses. In neutron star matter the presence of an appreciable electron chemical potential, μ_e , also change the conditions for CFL as discussed in the following.

In neutron star matter β -equilibrium relates the quark and electron chemical potentials

$$\mu_d = \mu_s = \mu_u + \mu_e. \quad (1)$$

Temperatures are normally much smaller than typical Fermi energies in neutron stars. If interactions are weak, the chemical potentials are then related to Fermi momenta by $\mu_i = \sqrt{m_i^2 + p_i^2}$. If the strange quark mass m_s is much smaller than the quark chemical potentials, Eq. (1) implies a difference between the quark Fermi momenta

$$p_u - p_d = \mu_e, \quad (2)$$

$$p_u - p_s \simeq \frac{m_s^2}{2\mu} - \mu_e, \quad (3)$$

$$p_d - p_s \simeq \frac{m_s^2}{2\mu}, \quad (4)$$

where μ is an average quark chemical potential. In Fig. 1 the difference between the Fermi momenta given by Eqs. (2-4) are plotted as function of electron chemical potential. Strange quark masses are estimated from low energy QCD $m_s \simeq 150 - 200$ MeV and typical quark

chemical potentials are $\mu \simeq 400 - 600$ MeV in quark matter [8]. Consequently, $m_s^2/2\mu \simeq 10 - 25$ MeV.

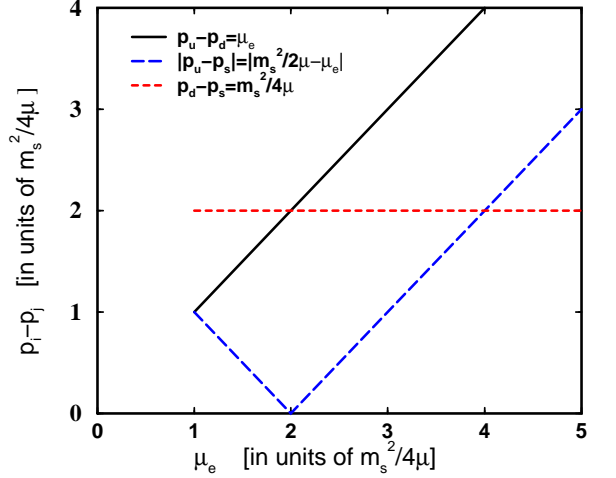


FIG. 1. The difference between Fermi momenta as function of electron chemical potentials. All quantities in units of $m_s^2/4\mu$. At densities where the difference exceeds the CSC gap the color-flavor locking is broken and CSC disappears.

Perturbative corrections change the relation between Fermi momenta and chemical potentials for relativistic quarks to $p_q = \mu_q(1 - 2\alpha_s/3\pi)^{1/3}$ and lead to corrections of order $\alpha_s m_s^2/\mu$ in Eqs. (3-4) for a massive strange quark. For weak coupling and small strange quark masses such effects are small and will be ignored in the following.

The BCS gap equation has previously been solved for u, d and u, d, s quark matter ignoring electrons and β -equilibrium and the conditions for condensates of dicolor pairs and CFL respectively were obtained [1–4]. The CFL condition consists of three pair-wise “CFL” conditions

$$\Delta > |p_i - p_j|, \quad i, j = u, d, s, \quad (5)$$

and thus require both small electron chemical potential and strange quark mass according to Eqs. (2-4). Alternatively, only one of these conditions may be fulfilled. For example, ud “CFL” requires small electron chemical potential whereas ds “CFL” requires a small strange quark mass. The us “CFL” condition can actually be satisfied when $\mu_e \simeq m_s^2/2\mu$. For these three cases a condensate of dicolor pairs (2CS) can appear between ud , us , ds -quarks analogous to the standard ud 2CS usually discussed for symmetric ud quark matter [2,3].

The magnitude of the electron chemical potential will now be discussed for electrically charge neutral bulk

quark matter as well as for mixed phases of quark and nuclear matter [7].

Bulk quark matter must be electrically neutral, i.e., the net positively charged quark density must be balanced by the electron density

$$n_e = \frac{\mu_e^3}{3\pi^2} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s \simeq \frac{1}{\pi^2} \left(\frac{1}{2}m_s^2\mu - 2\mu_e\mu^2 \right). \quad (6)$$

Muons will appear when their chemical potential exceeds their rest masses, $\mu_\mu = \mu_e > m_\mu$, but this occurs for very large electron chemical potentials only, where CSC is unlikely, and muons will therefore be ignored here.

When the electron chemical potential is small as compared to the quark chemical potentials the l.h.s. of Eq. (6) is negligible and we obtain

$$\mu_e \simeq \frac{m_s^2}{4\mu}. \quad (7)$$

Albeit the electron charge density is negligible, the electrons affect quark chemical potentials through β -equilibrium. The “CFL” condition for ud -quarks, Eq. (2), is therefore the *same* as the “CFL” condition for us -quarks, Eq. (3), in pure quark matter.

A mixed phase of quark and nuclear matter has lower energy per baryon at a wide range of densities [6] if the Coulomb and surface energies associated with the structures are sufficiently small [7,8]. The mixed phase will then consist of two coexisting phases of nuclear and quark matter in droplet, rod- or plate-like structures in a continuous background of electrons much like the mixed phase of nuclear matter and a neutron gas in the inner crust of neutron stars [9]. Another requirement for a mixed phase is that the length scales of such structures must be shorter than typical screening lengths.

In the mixed phase the nuclear and quark matter will be positively and negatively charged respectively. Total charge neutrality requires

$$n_e = (1-f)n_p + f\left(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s\right), \quad (8)$$

where n_p is the proton density and f is the “filling fraction”, i.e. the fraction of the volume filled by quark matter. For pure nuclear matter, $f = 0$, the nuclear symmetry energy can force the electron chemical potential above ~ 100 MeV at a few times normal nuclear matter densities. With increasing filling fraction, however, negative charged droplets of quark matter replace some of the electrons and μ_e decreases. With increasing density and filling fraction it drops to its minimum value given by Eq. (7) corresponding to pure quark matter, $f = 1$.

Gap sizes of order a few MeV or less were originally estimated within perturbative QCD [1]. Non-perturbative calculations give large gaps of order a few tens of MeV [2–5]. At high densities $\mu \rightarrow \infty$ and

weak couplings (g) the gap has been calculated, $\Delta \simeq g^{-5} \exp(-3\pi^2/\sqrt{2}g)$, and drops below ~ 1 MeV [10]. Information about the non-perturbative low density limit may obtain from studies of dilute Fermi systems. At low densities, when the typical range of interaction, R , is much shorter than the scattering length, $|a|$, the gap is $\Delta \sim \mu \exp(-\pi/2p_F|a|)$ [11], where a is the non-relativistic scattering length. For $p_F|a| \sim 1$ the gap may be large - of order the Fermi energy. Large scattering lengths appear when the two interacting particles almost form a bound state. However, confinement of quarks is different from such a simple bound state analogy and the large gap of order the chemical potential may not be conjectured for relativistic quarks.

In the mixed phase gaps may also be affected by the finite size of the quark matter structures. For example, nuclei pairing is dominated by surface effects [12] since gaps in nuclear matter are larger at lower densities. As droplets of quark matter are of similar size and baryon number we may expect similar finite size effects to enhance the CSC gap sizes.

For large gaps it may also be energetically favorable to have spatially varying quark chemical potentials and densities such that CFL occurs in some regions but not in others. From the gain in energy of order Δ^2/μ per particle the system must, however, pay Coulomb and surface energies associated with these structures [7]. A similar scenario is considered in [13] for u,d quark matter.

Consequences: Some bulk or mixed phase regions of quark matter in neutron stars can be color superconducting either by CFL or 2CS depending on the gap sizes, electron chemical potentials and strange quark masses as described above. Furthermore, temperatures in neutron stars are so low, $T \lesssim 10^6 K \simeq 10^{-4}$ MeV, that quark matter structures would be solid frozen. As a consequence, lattice vibration will couple electrons at the Fermi surface with opposite momenta and spins via phonons and lead to a “standard” BCS gap for electrons. The isotopic masses are similar but as densities and Debye frequencies are larger, we can expect considerably larger BCS gaps for electrons. At typical neutron star densities neutrons and protons are superfluid as well due to 1S_0 and, in the case of protons, also 3P_2 pairing [8]. These superfluid and superconducting components will have drastically different transport properties than normal Fermi liquids [14]. Generally the resistance, specific heat, viscosities, cooling, etc. are suppressed by factors of order $\sim \exp(-\Delta_i/T)$, where Δ_i is the gap of quarks, nucleons or electrons.

In relativistic nuclear collisions the strange quark chemical potential is zero initially and expansion times $\sim R/c \simeq 10$ fm/c are short as compared to time scales for weak decay and strangeness distillation. Therefore, $\mu_s \simeq 0$ and we expect no CFL. In heavy ion collisions the amount on neutrons and therefore also d -quarks exceed that of protons and u -quarks. The resulting difference, $|p_d - p_u|$, can prohibit a 2CS depending on density,

temperature and gap size.

In summary the conditions for color superconductivity in quark matter were given in Eqs.)2-4) for electrically charge neutral neutron star matter in β -equilibrium - both bulk quark matter and mixed phases of quark and nuclear matter. The electron chemical potential and strange quark mass affect the various quark chemical potentials. For CFL to occur the gap must exceed both the electron chemical potential, $\Delta \gtrsim \mu_e$, and the mismatch in Fermi momenta induced by a massive strange quark, $\Delta \gtrsim m_s^2/2\mu$. Alternatively, if μ_e , $m_s^2/2\mu$ or the difference $|\mu_e - m_s^2/2\mu|$ are smaller than the gap, then a condensate of dicolor pairs (2CS) can appear between ud , ds , us -quarks respectively analogous to the standard ud 2CS usually discussed for symmetric ud quark matter.

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